PROPAGATION OF A THERMAL WAVE IN A GAS HEATED BY INSTANTANEOUS MONOCHROMATIC RADIATION

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Reference [1] used an abiabatic approximation to examine self-similar motion of a gas heated by an instantaneous point isotropic source of monochromatic radiation. Reference [2] solved the analogous problem with a homothermic model, corresponding to the case of a strongly heated gas with substantial radiative conduction effects.

However, one must bear in mind that under strong heating the phase of gas motion precedes that of the thermal wave [3].

In this paper we consider the similarity laws for a thermal wave propagating through a region heated by an instantaneous pulse of monochromatic radiation.

Let the internal energy per unit volume of the heated gas at t = 0 satisfy the relation

$$E(r, 0) = A/r^2. \tag{1}$$

The initial state of the gas satisfying Eq. (1) can be obtained with an instantaneous liberation of energy E_0 in the cold gas of density ρ_0 in the form of monochromatic radiation with mean free path L [1-2]. In this case $E(r, 0) = E_0 e^{-r/L}/4\pi r^2$, and coincides with Eq. (1) in the limit $r \ll L$ ($A = E_0/4\pi L$).

We assume that for t > 0 a thermal wave propagates from the center of symmetry in the heated region, the wave resulting from the high temperatures near the center of symmetry. We note that the original state of the gas defined by Eq. (1) also allows propagation of a cooling wave converging towards the center, in addition to the thermal wave. In that case the problem is complex and loses its similarity.

However, the velocity of the cooling wave is small in the time interval considered, and to a first approximation one can neglect radiation from the heated region.

The equation describing the behavior of the thermal wave, in the radiative conduction approximation, can be written, in a spherical coordinate system, in the form [3]

$$\frac{\partial E}{\partial t} = a\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 E^n \frac{\partial E}{\partial r}\right). \tag{2}$$

We note that the internal energy per unit gas volume $E = \rho_0 \alpha T^{k+1}$ is used in Eq. (2) as the unknown function, instead of the temperature. This approximation gives a better description of the properties of a heated gas at high temperature, allowing for the specific heat of the gas C_V and the radiative mean free path λ_R to vary as a power of the temperature:

$$C_V = \partial E/\partial T = (1 + k)\alpha T^k,\ \lambda_R = BT^m.$$

Taking the above into account, the coefficients a and n in Eq. (2) have the values

$$a = \frac{16\sigma B}{3(1+k)(\rho_0\alpha)^{(4+m)/(1+k)}},\quad n = \frac{m+3-k}{1+k}.$$

The solution of Eq. (2) must satisfy the following conditions.

At the center of symmetry, because there are no heat sources, the radiative flux is zero:

$$\lim_{r\to 0} r^2 \frac{\partial E}{\partial r} = 0. \tag{3}$$

We can neglect the influence of radiation at large distance.

$$\lim_{r\to\infty} E(t, r) = A/r^2. \tag{4}$$

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In addition, we apply the integral energy conservation law to the heated gas:

$$\lim_{r\to\infty}\int_0^r E(t, r)\, r^2 dr = Ar. \tag{5}$$

The solution of Eq. (2) accounting for Eqs. (3)-(5) is the similarity problem.

We introduce the similarity variables:

$$x = r/(A^n at)^{1/2(n+1)},\ f = E(aA^{-1}t)^{1/(n+1)}. \tag{6}$$

We substitute Eq. (6) into Eqs. (2)-(5). We obtain an ordinary second-order differential equation

$$2(n+1)\frac{1}{x^2}\frac{d}{dx}\left(x^2 f^n \frac{df}{dx}\right) + x\frac{df}{dx} + 2f = 0. \tag{7}$$

Correspondingly, we transform the boundary conditions

$$\lim_{x\to 0} x^2\frac{df}{dx} = 0,\quad \lim_{x\to\infty} f(x) = 1/x^2,\quad \lim_{x\to\infty}\int_0^x f(\xi)\,\xi^2 d\xi = x. \tag{8}$$

Integrating Eq. (7), we have

$$2x\frac{d}{dx}(f^{n+1}) + 2f^{n+1} + x^2 f = C \tag{9}$$

or, taking account of Eq. (8),

$$f(x) = \left[\frac{C}{2} - \frac{1}{2x}\int_0^x f(\xi)\,\xi^2 d\xi\right]^{1/(1+n)}.$$

Using condition (8), we find the value of the constant to be C = 1.

Thus, we obtain the first-order differential equation

$$\frac{df}{dx} = \frac{1 - x^2 f - 2f^{n+1}}{2(n+1)xf^n} \tag{10}$$

with the initial condition

$$f(0) = (1/2)^{1/(1+n)}. \tag{11}$$

We integrated Eq. (10) numerically, taking account of Eq. (11), up to values of x satisfying the condition

$$|f(x) - 1/x^2| < \varepsilon. \tag{12}$$

The accuracy of the solution was monitored by inspecting the condition

$$\left|\int_0^x f(\xi)\,\xi^2 d\xi - x\right| < \varepsilon_1. \tag{13}$$

The solid line in Fig. 1 shows the dependence of the function f(x) for n = 3, and the broken line shows the relation $f_1 = 1/x^2$ corresponding to the original state of the heated gas. It can be seen that condition (12) is satisfied for $x \geqslant 2.41$ for $\varepsilon \leqslant 0.01$. The integration step satisfying Eq. (13) for $\varepsilon_1 = 0.005$ was h = 0.001.

The radius of the thermal wave front is given by

$$r_f = x_f\,(A^n\, at)^{1/2(n+1)}, \tag{14}$$

where we can take $x_f = 2.41$ as the value of x_f.

It is interesting to compare the relations obtained with the thermal wave similarity solution for the case of a point liberation of energy E_0 [3]:

$$r_f = x_f\,[a(E_0/\rho_0 C_V)^n\, t]^{1/(3n+2)}.$$

For n = 3, according to Eq. (14), we have

$$r_f \sim t^{1/8},\ dr_f/dt \sim t^{-7/8},$$

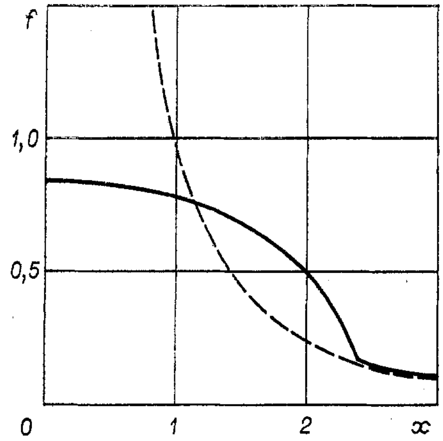

Fig. 1

while for the point liberation of energy we have $r_f \sim t^{1/11}$ and $dr_f/dt \sim t^{-10/11}$.

Thus, we have shown that the initial spatial distribution of released energy has an appreciable influence on the propagation of the thermal wave.

In conclusion, we note that later stages in the process, when the velocity of the thermal wave front is comparable with that of sound in the heated gas, can be described by the methods examined in [1, 2].

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RADIATIVE—CONDUCTIVE HEAT TRANSFER IN A THIN SEMITRANSPARENT PLATE IN THE GUIDED-WAVE APPROXIMATION FOR A TEMPERATURE- AND FREQUENCY-DEPENDENT ABSORPTION COEFFICIENT

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The radiative—conductive heat-transfer problem has been studied previously [1] for a thin semitransparent cylinder whose refractive index is much greater than unity. This class of problems arises in the investigation of the temperature fields in semitransparent crystals such as sapphire or lithium niobate during pulling from the melt by the Czochralski or Stepanov method. It is shown that the radiative energy transfer in the indicated cylinder can be described by the so-called guided-wave approximation, where only those rays which undergo total internal reflection at the boundary of the cylinder are included in the radiant flux in its interior. A comparison of the analytical with experimental results and a study of heat transfer in a semitransparent cylinder coated with a thin absorbing film are reported in [2]. Heat transfer in a thin infinitely wide semitransparent plate is discussed in the same paper. However, the constant absorption coefficient postulated in [1, 2] appears to be rather crude. For example, according to the data of [3], the absorption coefficient of sapphire in the temperature range 1200-2000°C varies quite considerably, from 0.004 to 0.5 cm^{-1}, in the interval of wavelengths up to 4 μm, where the bulk of the radiated energy is concentrated. It is important, therefore, to calculate the temperature field in a semitransparent plate whose absorption coefficient depends on the temperature and frequency.

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